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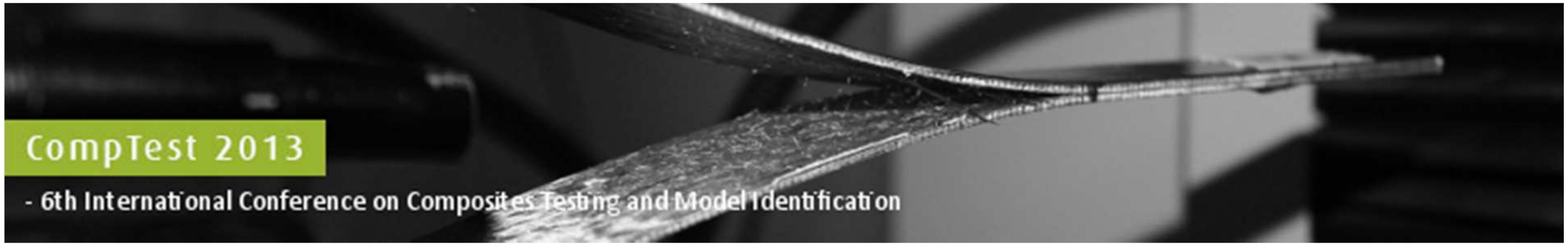

Explicit expressions for the crack length correction parameters for the DCB, ENF, and MMB tests on multidirectional laminates

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Standard mode I and mode II delamination tests

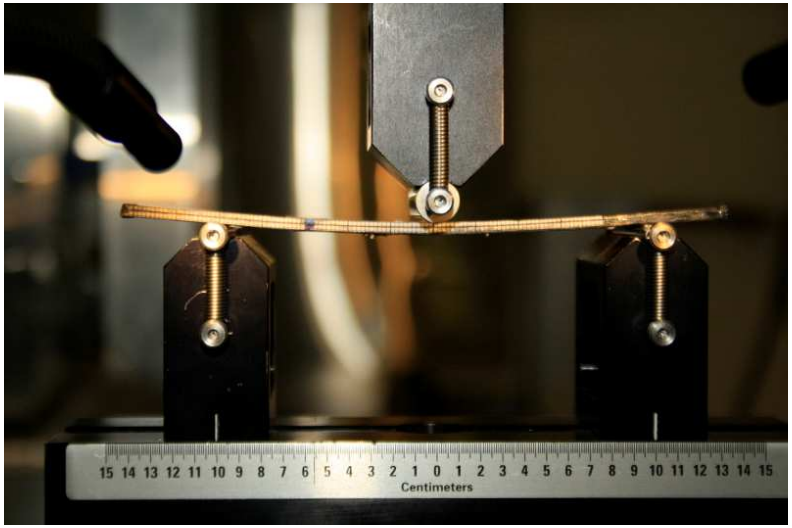
Double cantilever beam (DCB)

AECMA prEN 6033:1995: *Determination of interlaminar fracture toughness energy. Mode I* G_{Ic}.

ISO 15024:2001: *Determination of mode I interlaminar fracture toughness,* G_{Ic}*, for unidirectionally reinforced materials*.

ASTM D5528-01(2007)e3: *Standard Test Method for Mode I Interlaminar Fracture Toughness of Unidirectional Fiber-Reinforced Polymer Matrix Composites.*

End notched flexure (ENF)

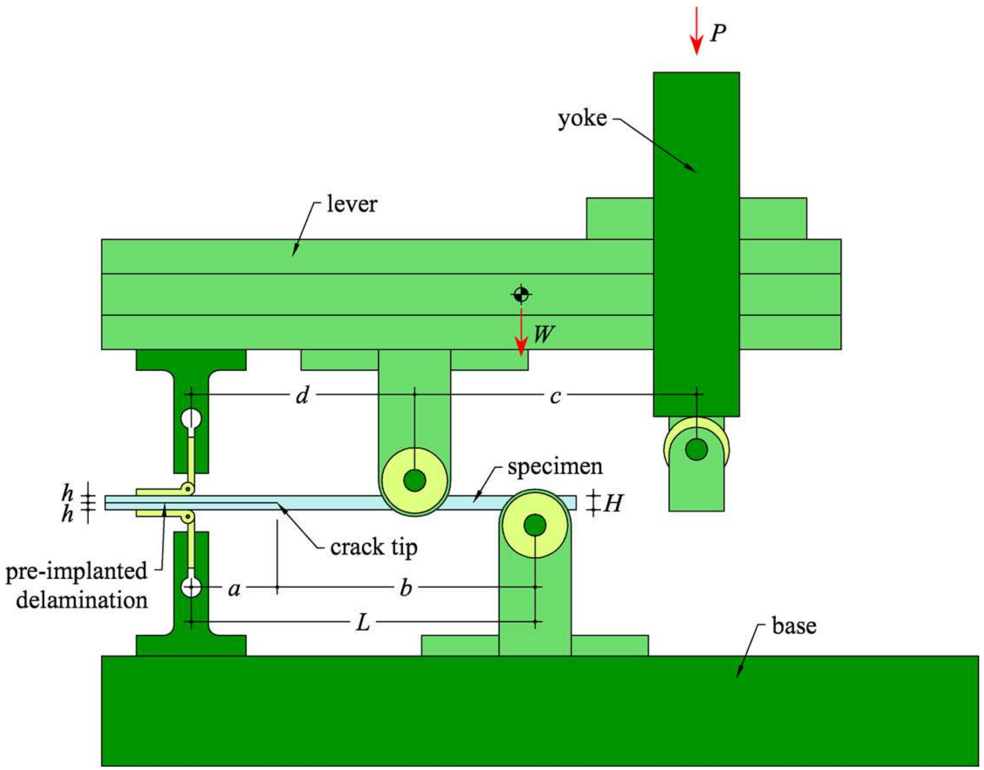


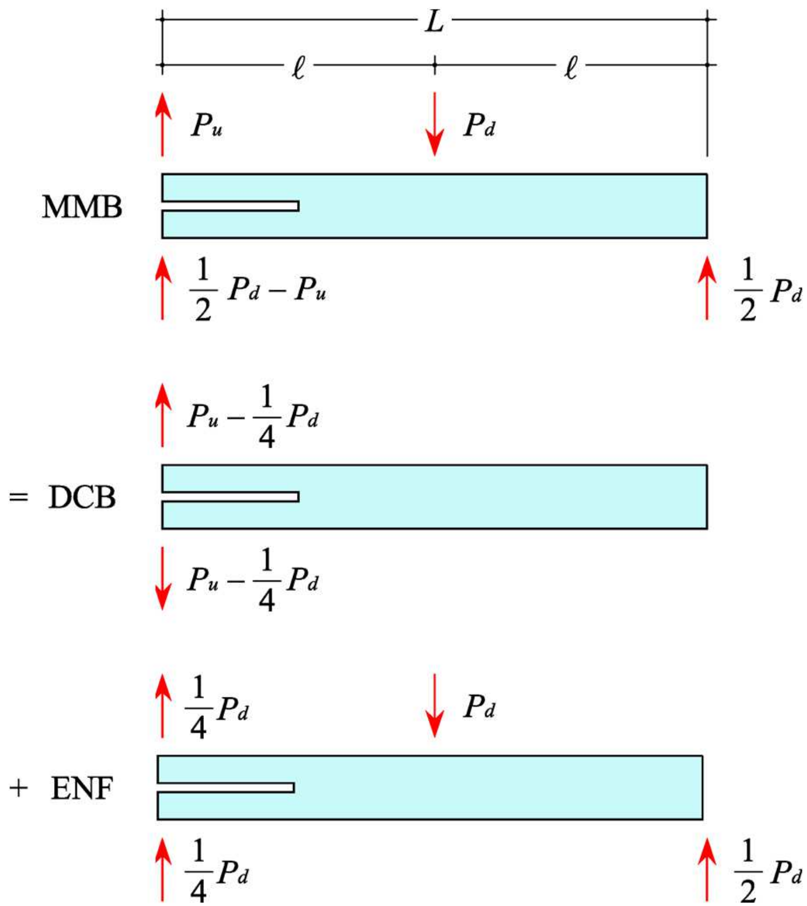
JIS K 7086-1993: *Testing methods for interlaminar fracture toughness of carbon fibre reinforced plastics*.

AECMA prEN 6034:1995: *Determination of interlaminar fracture toughness energy. Mode II* G_{IIc}.

Standard I/II mixed-mode delamination test

Mixed-mode bending (MMB)

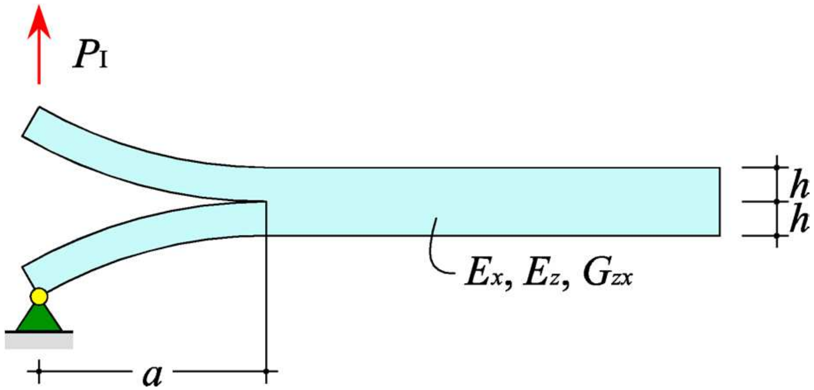




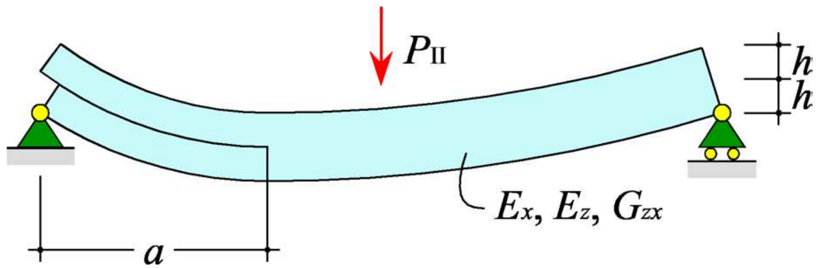
ASTM D6671/D6671M-06: *Standard Test Method for Mixed Mode I-Mode II Interlaminar Fracture Toughness of Unidirectional Fiber Reinforced Polymer Matrix Composites*.

Simple beam theory (SBT) model

Double cantilever beam (DCB)



End notched flexure (ENF)



Mode I energy release rate

$$G_{\mathrm{I}}^{\mathrm{SBT}} = \frac{12 P_{\mathrm{I}}^2 a^2}{B^2 E_x h^3}$$

Specimen's compliance

$$C_{\mathrm{DCB}}^{\mathrm{SBT}} = \frac{8a^3}{B E_x h^3}$$

Mode II energy release rate

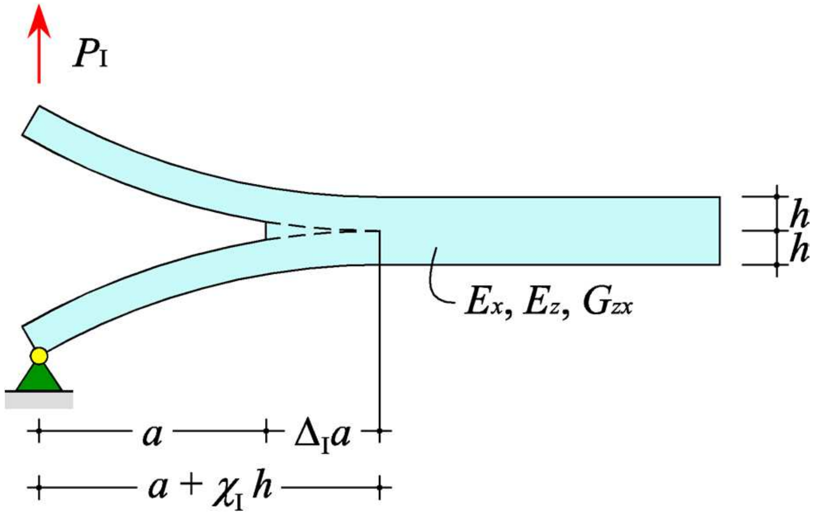
$$G_{\mathrm{II}}^{\mathrm{SBT}} = \frac{9 P_{\mathrm{II}}^2 a^2}{16 B^2 E_x h^3}$$

Specimen's compliance

$$C_{\mathrm{ENF}}^{\mathrm{SBT}} = \frac{3a^3 + 2\ell^3}{8 B E_x h^3}$$

Corrected beam theory (CBT) model

Double cantilever beam (DCB)



Mode I energy release rate

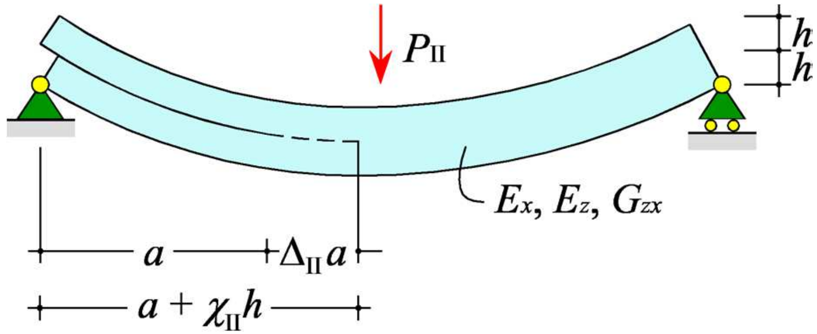
$$G_I^{CBT} = \frac{12P_I^2}{B^2 E_x h^3}(a + \chi_I h)^2$$

Mode I crack length correction parameter

$$\chi_I = \sqrt{\frac{E_x}{11\,G_{zx}}[3 - 2(\frac{\Gamma}{1+\Gamma})^2]}$$

$$\text{where } \Gamma = 1.18\sqrt{E_x E_z} / G_{zx}$$

End notched flexure (ENF)



Mode II energy release rate

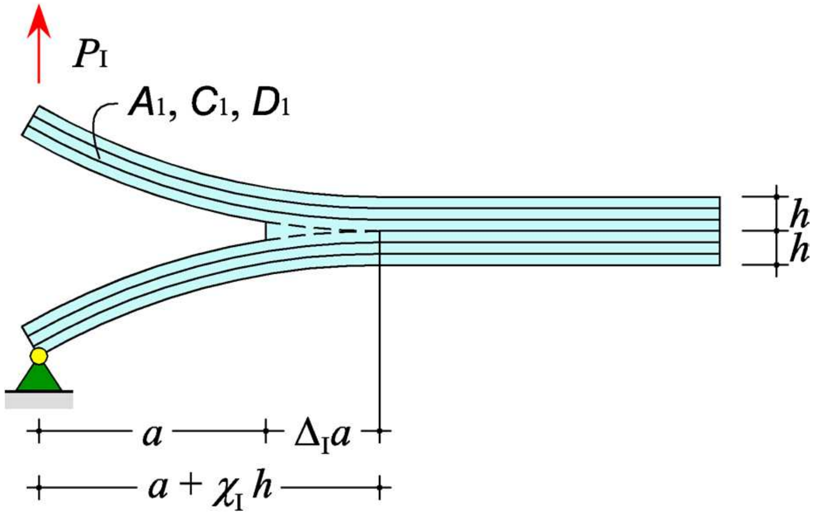
$$G_{II}^{CBT} = \frac{9P_{II}^2}{16B^2 E_x h^3}(a + \chi_{II} h)^2$$

Mode II crack length correction parameter

$$\chi_{II} = 0.42\chi_I$$

Laminated specimens

Double cantilever beam (DCB)



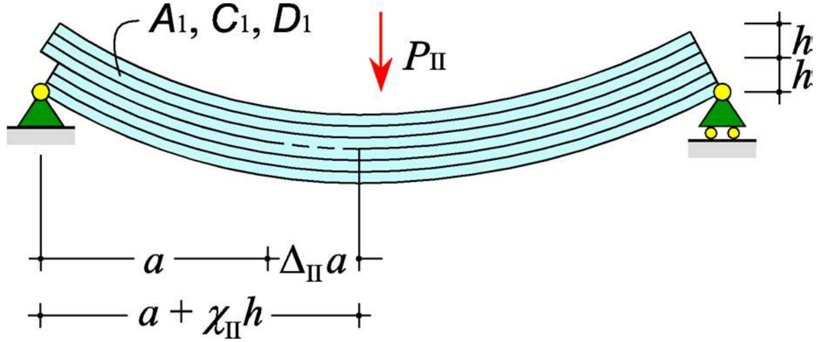
Mode I energy release rate

$$G_{\mathrm{I}}^{\mathrm{CBT}} = \frac{P_{\mathrm{I}}^2}{B^2 D_1}(a + \chi_{\mathrm{I}} h)^2$$

Mode I crack length correction parameter

$$\chi_{\mathrm{I}} = ?$$

End notched flexure (ENF)



Mode II energy release rate

$$G_{\mathrm{II}}^{\mathrm{CBT}} = \frac{P_{\mathrm{II}}^2}{16 B^2 D_1} \frac{A_1 h^2}{A_1 h^2 + 4 D_1}(a + \chi_{\mathrm{II}} h)^2$$

Mode II crack length correction parameter

$$\chi_{\mathrm{II}} = ?$$

Enhanced beam theory (EBT) model

Mixed-mode bending (MMB)

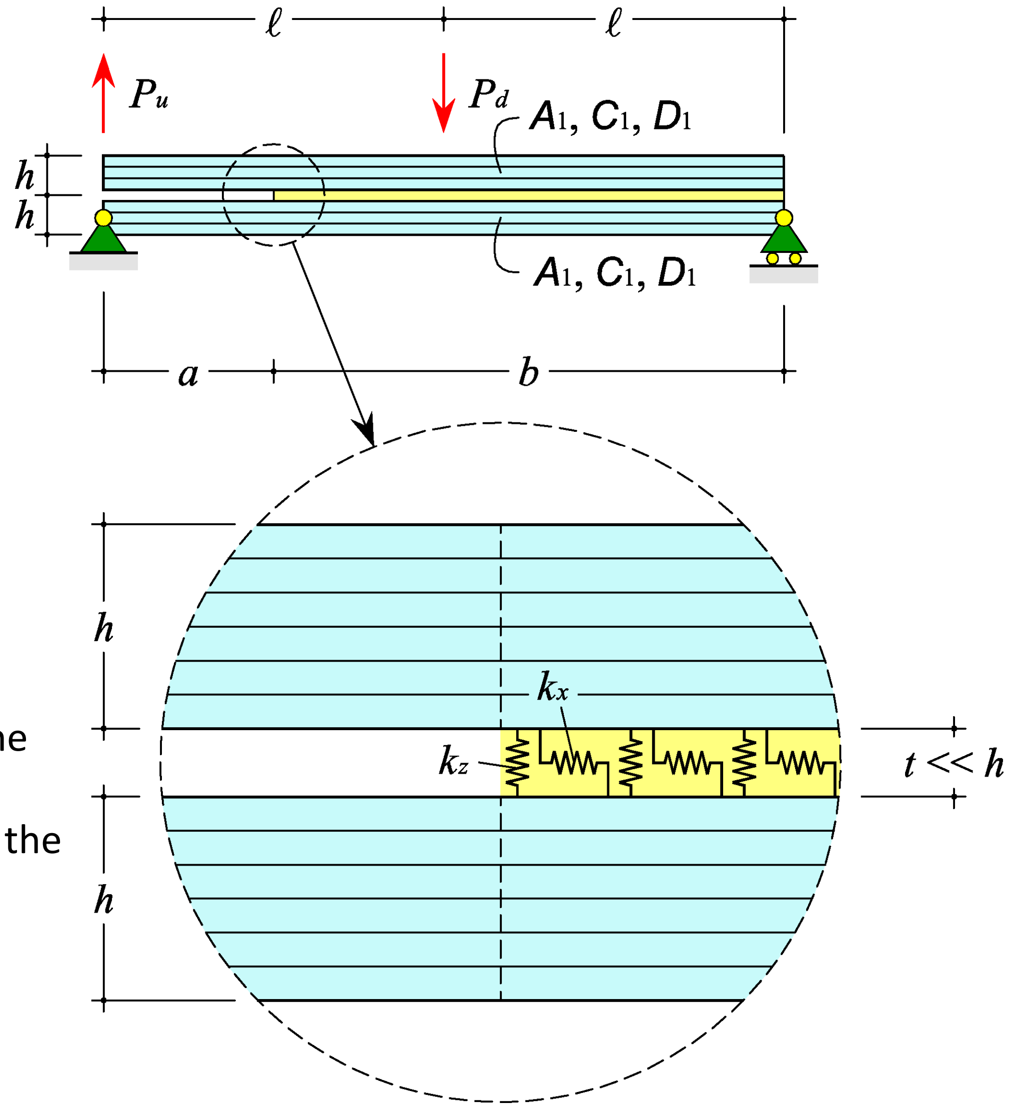
Hypotheses:

i) specimens split into two sublaminates having same extensional, shear, and bending stiffnesses;

ii) general stacking sequence allowed, but no shear-extension and no bending-extension coupling;

iii) sublaminates connected by an elastic interface, which transmits both normal and tangential stresses;

iv) negligible non-linear effects.



Results:

i) complete, exact analytical solution to the differential problem;

ii) simplified, approximate expressions for the specimen's compliance, energy release rate, and mode mixity;

iii) solutions for the DCB and ENF tests are obtained as special cases.

Enhanced beam theory (EBT) model

Exact analytical solution

Mode I and II energy release rates

$$G_{\mathrm{I}}^{\mathrm{EBT}} = \frac{\sigma_0^2}{2k_z}, \quad G_{\mathrm{II}}^{\mathrm{EBT}} = \frac{\tau_0^2}{2k_x}$$

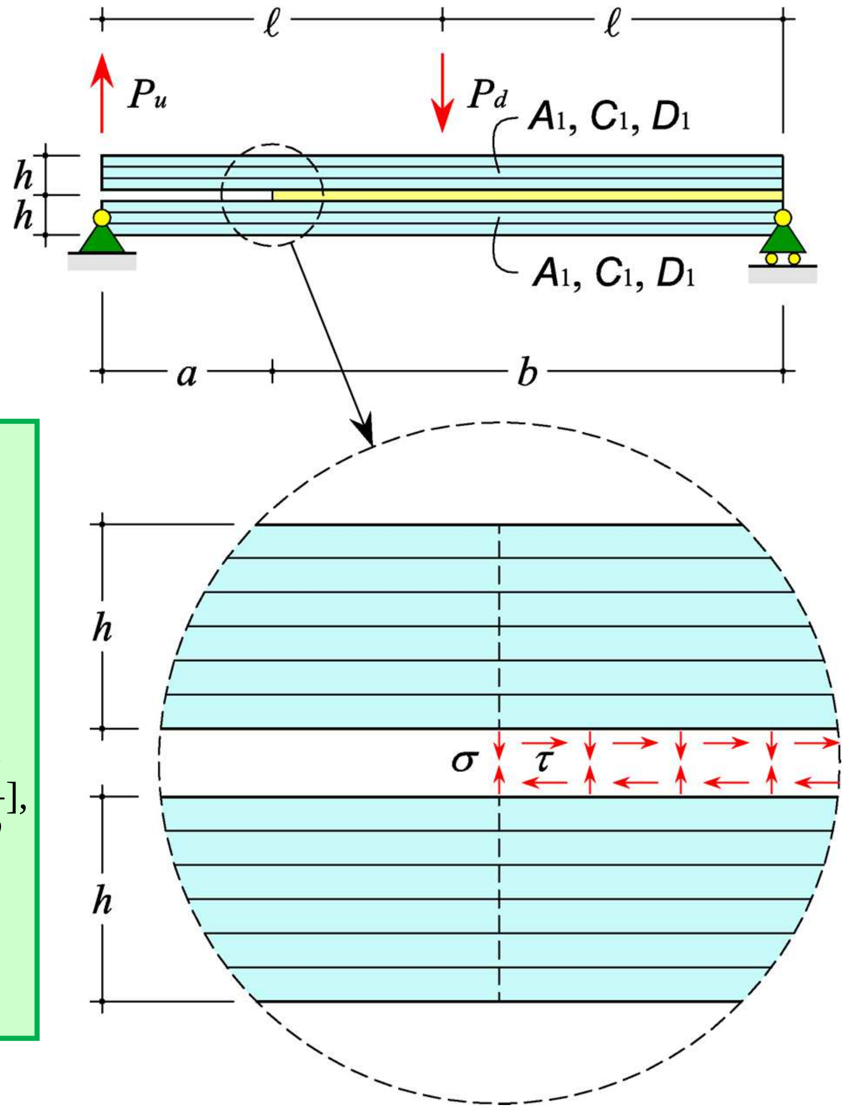
Interfacial stresses at the crack tip

$$\sigma_0 = \frac{P_{\mathrm{I}}}{B}\Big[\frac{(\lambda_1^2 - \lambda_2^2)(\lambda_1 \tanh\lambda_2 b - \lambda_2 \tanh\lambda_1 b)}{D} + \lambda_1\lambda_2 a \frac{(\lambda_1^2 + \lambda_2^2)(1 - \mathrm{sech}\,\lambda_1 b\,\mathrm{sech}\,\lambda_2 b)}{D} - \lambda_1\lambda_2 a \frac{2\lambda_1\lambda_2 \tanh\lambda_1 b \tanh\lambda_2 b}{D}\Big],$$

$$\tau_0 = \frac{P_{\mathrm{II}}}{Bh}\frac{A_1 h^2}{A_1 h^2 + 4D_1}\Big[\frac{1}{2}(1 + \lambda_5 a \coth\lambda_5 b) - \frac{\sinh\lambda_5 \ell}{\sinh\lambda_5 b}\Big],$$

where

$$D = (\lambda_1^2 + \lambda_2^2)\tanh\lambda_1 b \tanh\lambda_2 b - 2\lambda_1\lambda_2(1 - \mathrm{sech}\,\lambda_1 b\,\mathrm{sech}\,\lambda_2 b)$$



Enhanced beam theory (EBT) model

Approximate expressions

Mode I and II energy release rates

$$G_{\mathrm{I}}^{\mathrm{EBT}} \cong \frac{P_{\mathrm{I}}^2}{B^2 D_1}\left(a+\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)^2$$

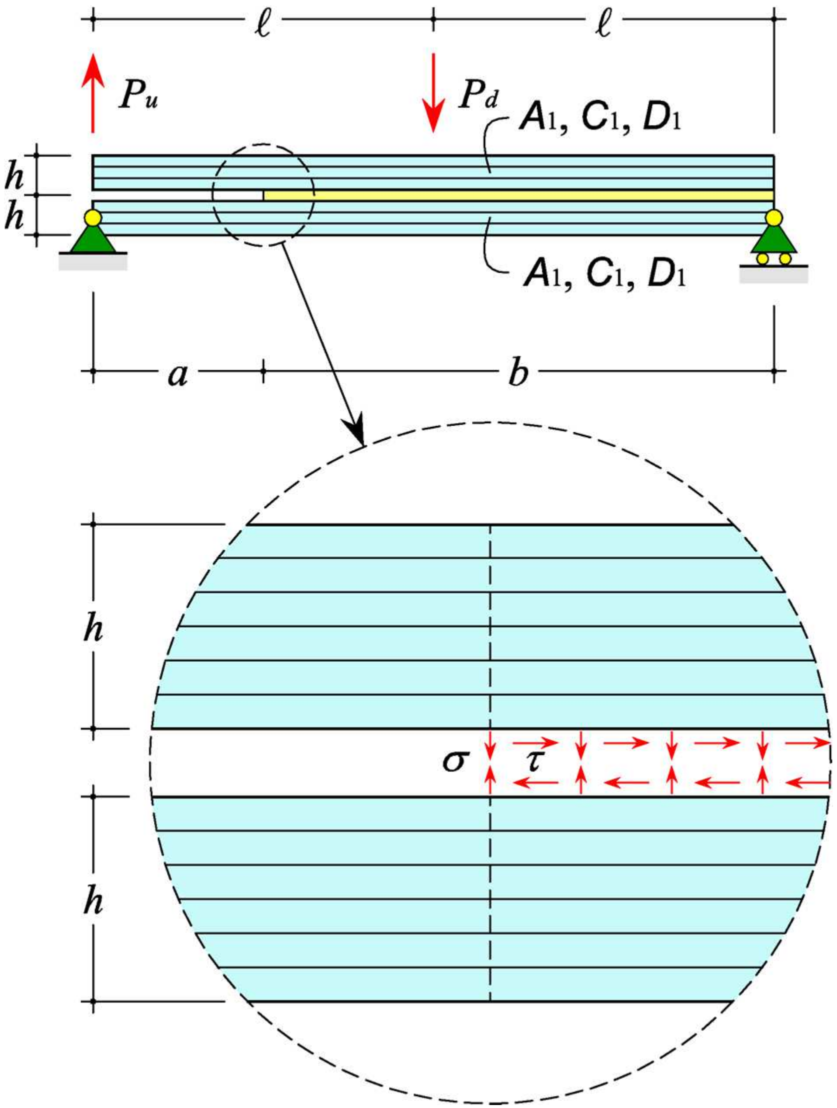
$$G_{\mathrm{II}}^{\mathrm{EBT}} \cong \frac{P_{\mathrm{II}}^2}{16 B^2 D_1}\frac{A_1 h^2}{A_1 h^2 + 4D_1}\left(a+\frac{1}{\lambda_5}\right)^2$$

Roots of the characteristic equations of the governing differential equations

$$\lambda_1 = \sqrt{\frac{k_z}{C_1}\left(1+\sqrt{1-\frac{2C_1^2}{k_z D_1}}\right)}$$

$$\lambda_2 = \sqrt{\frac{k_z}{C_1}\left(1-\sqrt{1-\frac{2C_1^2}{k_z D_1}}\right)}$$

$$\lambda_5 = \sqrt{2k_x\left(\frac{1}{A_1}+\frac{h^2}{4D_1}\right)}$$



Enhanced beam theory (EBT) model

Crack length correction parameters

Mode I and II energy release rates

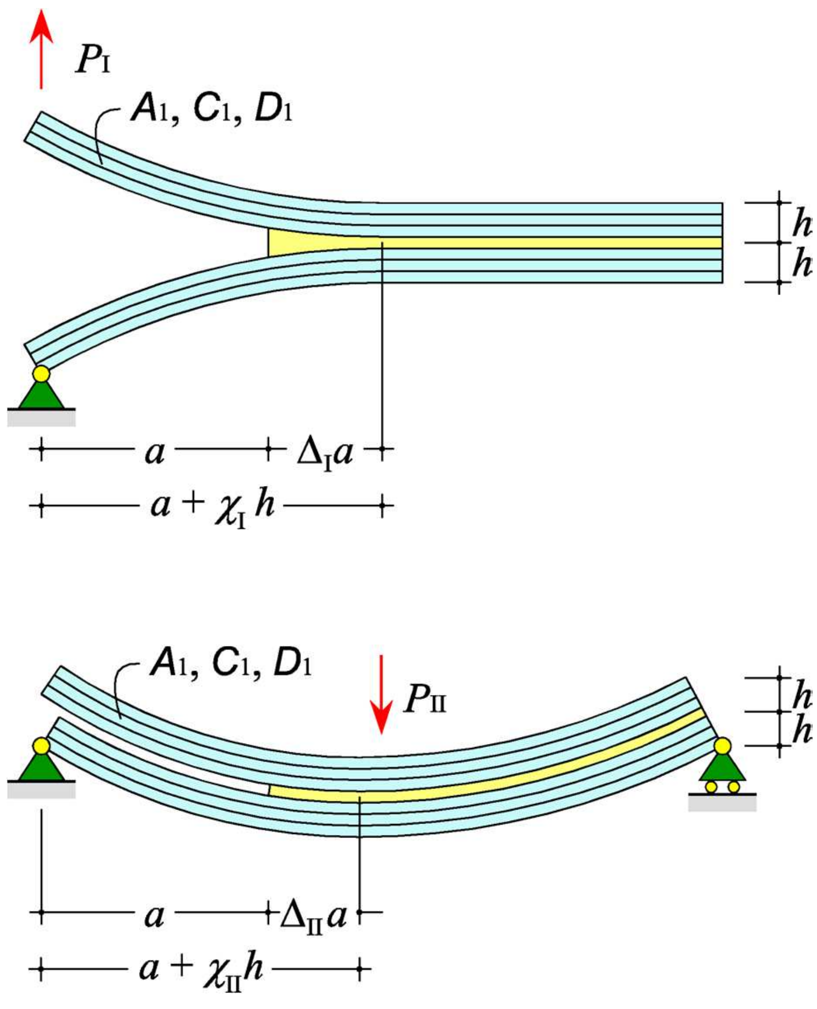
$$G_{\mathrm{I}}^{\mathrm{EBT}} \cong \frac{P_{\mathrm{I}}^2}{B^2 D_1}(a+\chi_{\mathrm{I}} h)^2$$

$$G_{\mathrm{II}}^{\mathrm{EBT}} \cong \frac{P_{\mathrm{II}}^2}{16 B^2 D_1}\frac{A_1 h^2}{A_1 h^2 + 4 D_1}(a+\chi_{\mathrm{II}} h)^2$$

Crack length correction parameters

$$\chi_{\mathrm{I}} = \frac{1}{h}\sqrt{\frac{D_1}{C_1} + \sqrt{\frac{2D_1}{k_z}}}$$

$$\chi_{\mathrm{II}} = \frac{1}{h}\frac{1}{\sqrt{2k_x\left(\frac{1}{A_1} + \frac{h^2}{4D_1}\right)}}$$



Application: unidirectional (UD) specimens

Carbon/PEEK composite (Reeder and Crews, 1992)

Specimen sizes

$$L = 100\text{ mm},\quad B = 25.4\text{ mm},\quad H = 2h = 3\text{ mm}$$

Ply elastic constants

$$E_x = 129\text{ GPa},\quad E_y = E_z = 10.1\text{ GPa},\quad G_{zx} = 5.5\text{ GPa}$$

Interface elastic constants

$$k_x = 31550\text{ N/mm}^3,\quad k_z = 23150\text{ N/mm}^3$$

Stacking sequence

$$[0_{12} // 0_{12}]$$

Crack length correction parameters according to CBT model

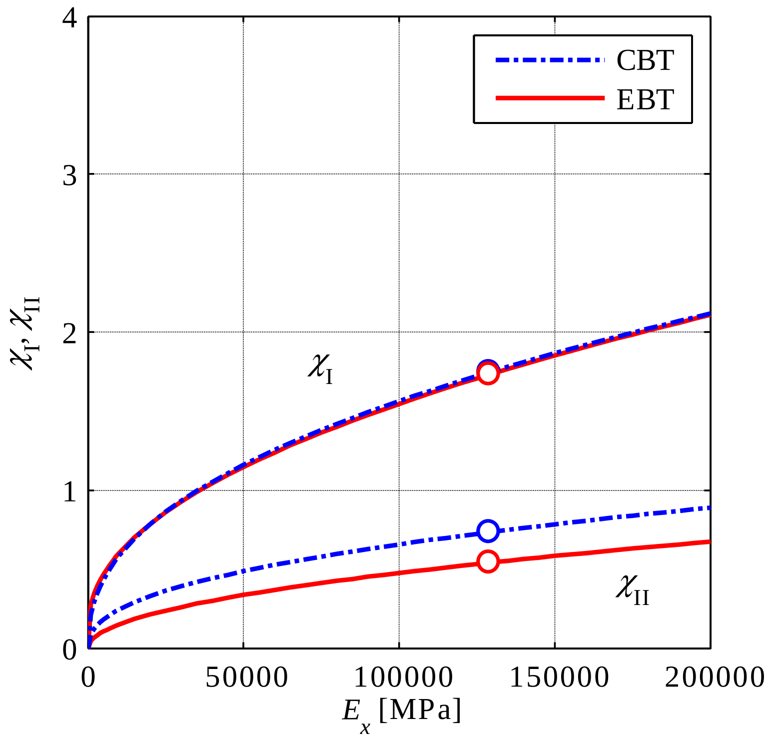
$$\chi_{\mathrm{I}} = 1.747,\quad \chi_{\mathrm{II}} = 0.734$$

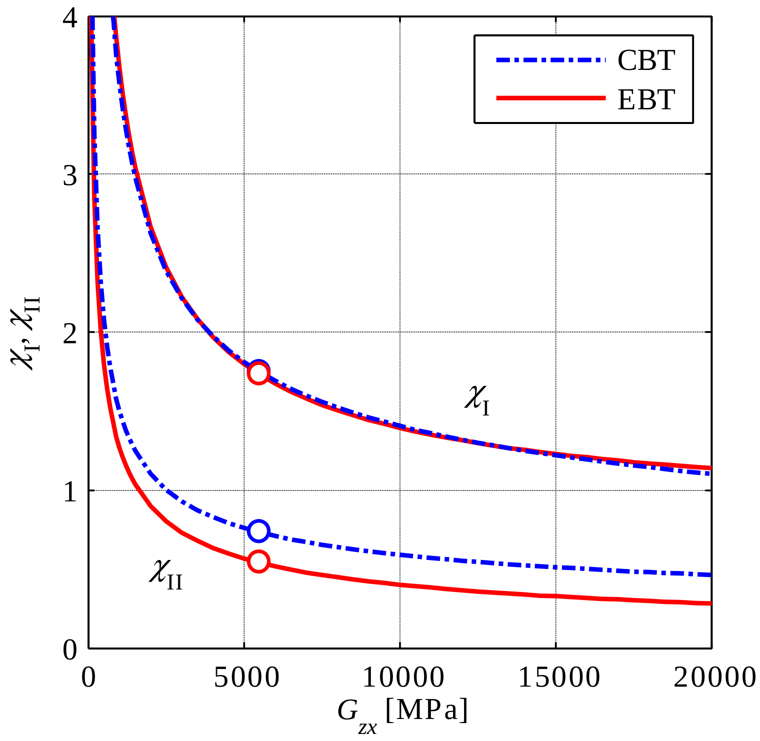
Crack length correction parameters according to EBT model

$$\chi_{\mathrm{I}} = 1.731,\quad \chi_{\mathrm{II}} = 0.541$$

Application: unidirectional (UD) specimens

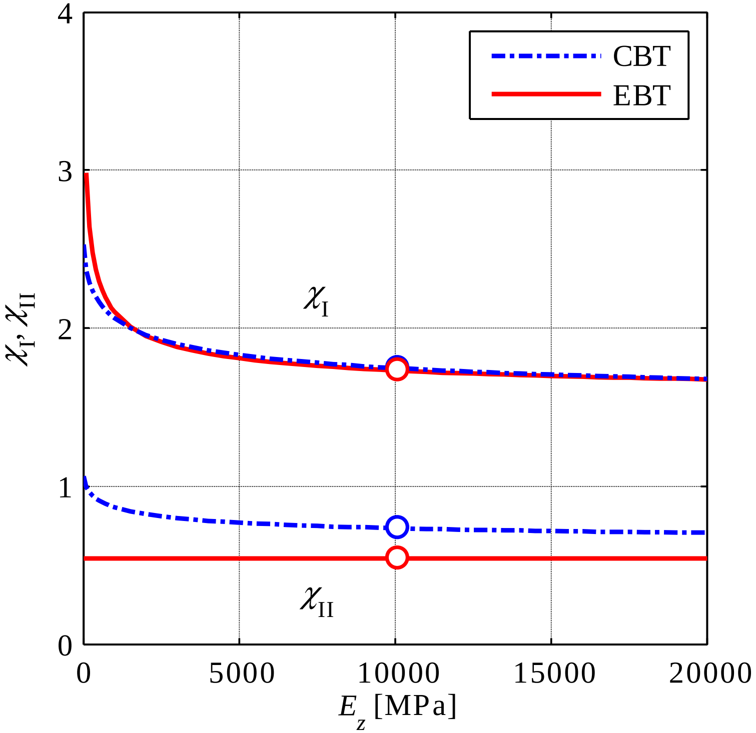
Comparison between CBT and EBT models





Application: unidirectional (UD) specimens

Comparison between CBT and EBT models



Application: multidirectional (MD) specimens

Glass/epoxy composite (Pereira & de Morais, 2006)

Specimen sizes

$$L = 100 \text{ mm}, \quad B = 20 \text{ mm}, \quad H = 2h = 6 \text{ mm}$$

Ply elastic constants

$$E_x = 33 \text{ GPa}, \quad E_y = 19 \text{ GPa}, \quad E_z = 8 \text{ GPa}, \quad G_{zx} = 4.8 \text{ GPa}$$

Interface elastic constants

$$k_x = 6147 \text{ N/mm}^3, \quad k_z = 4578 \text{ N/mm}^3$$

Stacking sequence

$$[(0_2/90)_6/0_2//(0_2/90)_6/0_2]$$

Sublaminate extensional, shear, and bending stiffnesses

$$A_1 = 86400 \text{ N/mm}, \quad C_1 = 10170 \text{ N/mm}, \quad D_1 = 66785 \text{ Nmm}$$

Crack length correction parameters according to EBT model

$$\chi_{\text{I}} = 1.153, \quad \chi_{\text{II}} = 0.541$$

Application: multidirectional (MD) specimens

Carbon/epoxy composite (Pereira & de Morais, 2008)

Specimen sizes

$$L = 100\ \text{mm}, \quad B = 20\ \text{mm}, \quad H = 2h = 6\ \text{mm}$$

Ply elastic constants

$$E_x = 130\ \text{GPa}, \quad E_y = E_z = 8.2\ \text{GPa}, \quad G_{zx} = 4.1\ \text{GPa}$$

Interface elastic constants

$$k_x = 12735\ \text{N/mm}^3, \quad k_z = 7765\ \text{N/mm}^3$$

Stacking sequence

$$[(0_2/90)_6/0_2//(0_2/90)_6/0_2]$$

Sublaminate extensional, shear, and bending stiffnesses

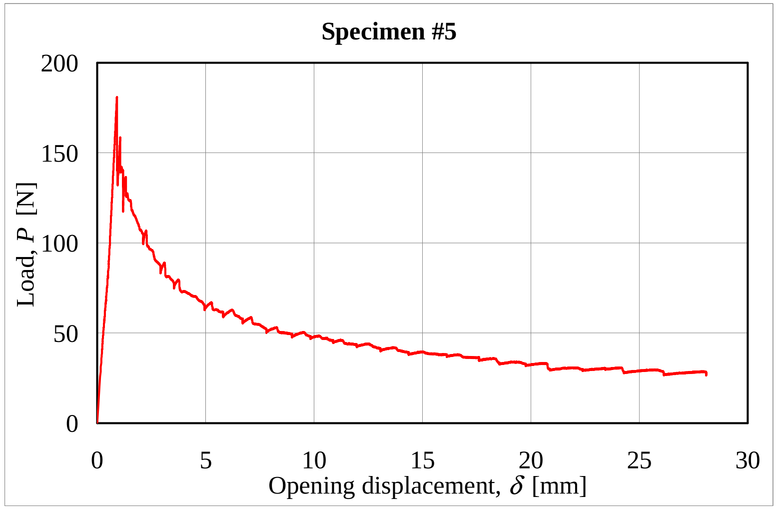
$$A_1 = 280380\ \text{N/mm}, \quad C_1 = 9130\ \text{N/mm}, \quad D_1 = 227550\ \text{Nmm}$$

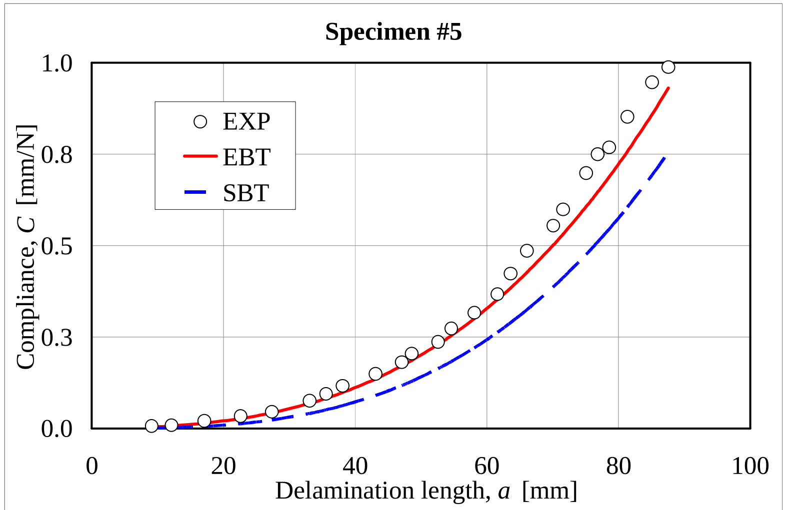
Crack length correction parameters according to EBT model

$$\chi_{\text{I}} = 1.903, \quad \chi_{\text{II}} = 0.569$$

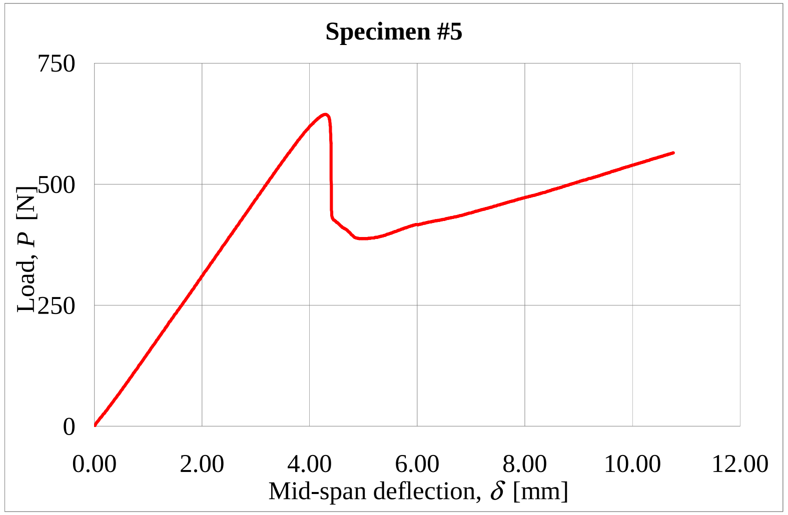
Experimental validation (work in progress)

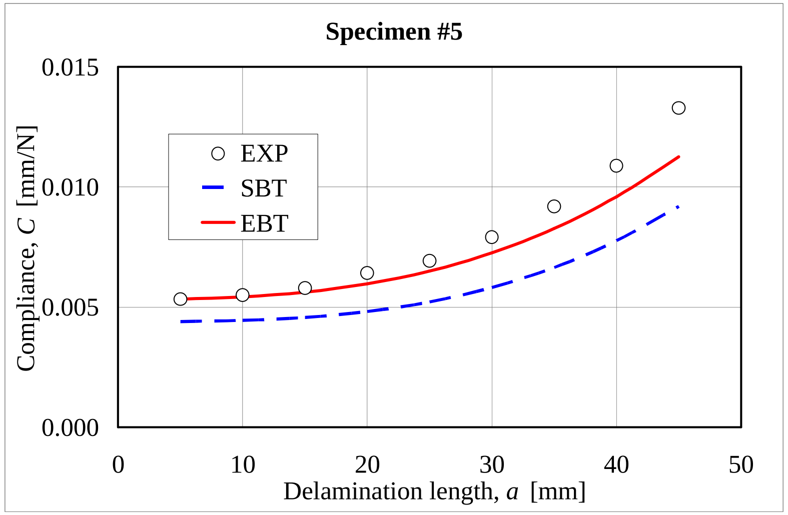
Double cantilever beam (DCB)





End notched flexure (ENF)





References

On the EBT model of the mixed-mode bending test

BENNATI, Stefano; FISICARO, Paolo; VALVO, Paolo Sebastiano (2013): An enhanced beam-theory model of the mixed-mode bending (MMB) test - Part I: literature review and mechanical model, *Meccanica*, **48** (2), p. 443-462. URL: http://dx.doi.org/10.1007/s11012-012-9686-3 (Erratum: http://dx.doi.org/10.1007/s11012-013-9697-8).

BENNATI, Stefano; FISICARO, Paolo; VALVO, Paolo Sebastiano (2013): An enhanced beam-theory model of the mixed-mode bending (MMB) test - Part II: applications and results, *Meccanica*, **48** (2), p. 465-484. URL: http://dx.doi.org/10.1007/s11012-012-9682-7 (Erratum: http://dx.doi.org/10.1007/s11012-013-9696-9).

On the estimation of the elastic interface constants

BENNATI, Stefano; VALVO, Paolo Sebastiano (2013): An experimental compliance calibration strategy for estimating the elastic interface constants of delamination test specimens, *AIMETA 2013 – XXI Congresso Nazionale dell'Associazione Italiana di Meccanica Teorica e Applicata* (Turin, Italy, September 17–20, 2013). URL: http://www.aimetatorino2013.it.

VALVO, Paolo Sebastiano; CORNETTI, Pietro (2013): Energetic estimation of the elastic interface constants for delamination modelling, *AIMETA 2013 – XXI Congresso Nazionale dell'Associazione Italiana di Meccanica Teorica e Applicata* (Turin, Italy, September 17–20, 2013). URL: http://www.aimetatorino2013.it.